Joint Optimization of Time-of-day Intervals and Robust Signal Timing for Isolated Intersection

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Bi-level Optimization Framework

$$\min_{g_P^l, C_P, L_P} \; Td = \sum_{n=1}^{D}\sum_{l=1}^{L}\sum_{P=1}^{\rho} d_P^n(g_P^l, C_P, L_P)$$

$$s.t.\,(UL)\quad \sum_{P=1}^{\rho} L_P = TL,\quad L_P^{Min} \le L_P \le L_P^{Max},\quad mod(L_P, C_P) = 0$$

$$\min_{g_P^l, C_P} \sum_{l=1}^{L}\sum_{P=1}^{\rho} \overline{d_P^n}(g_P^l, C_P) + \gamma \sigma_{d_P^n}(g_P^l, C_P)$$

$$s.t.\,(LL)\quad \sum_{l \in l^c} g_P^l + L_t = C_P,\quad g_P^{Min} \le g_P \le g_P^{Max},\quad C_P^{Min} \le C_P \le C_P^{Max}$$

Notations

Td	total delay for all TOD intervals on D days (s)
l	lane group (L lane groups in total)
l^c	set of critical lane groups
n	day (D days in total)
P	TOD interval (ρ TOD intervals in total)
d_P^n	total delay of TOD interval P on day n (s)
$\overline{d_P^n}$	average total delay of TOD interval P for D days (s)
$\sigma_{d_P^n}$	SD of total delay of TOD interval P for D days (s)
g_P^l	effective green time of lane group l in TOD interval P (s)
C_P	cycle length in TOD interval P (s)
L_P	duration of TOD interval P (s)
TL	sum of the durations of all TOD intervals (s)
L_t	total lost time for each cycle (s)
$L_P^{Min}(L_P^{Max})$	minimum (maximum) duration of TOD interval (s)
$g_P^{Min}(g_P^{Max})$	minimum (maximum) effective green time (s)
$C_P^{Min}(C_P^{Max})$	minimum (maximum) cycle length (s)
γ	robustness ratio

Algorithm 1: Method of adjusting TOD intervals for estimating delays

Input: Total No. of data-collecting periods I (with each duration of T_I), duration set of TOI intervals $\{L_P\}_{P=1}^{\rho}$, cumulative arrivals A_2, effective green time set $\{g_P\}_{P=1}^{\rho}$, effective red time before green set $\{r_P\}_{P=1}^{\rho}$, and cycle length set $\{C_P\}_{P=1}^{\rho}$.

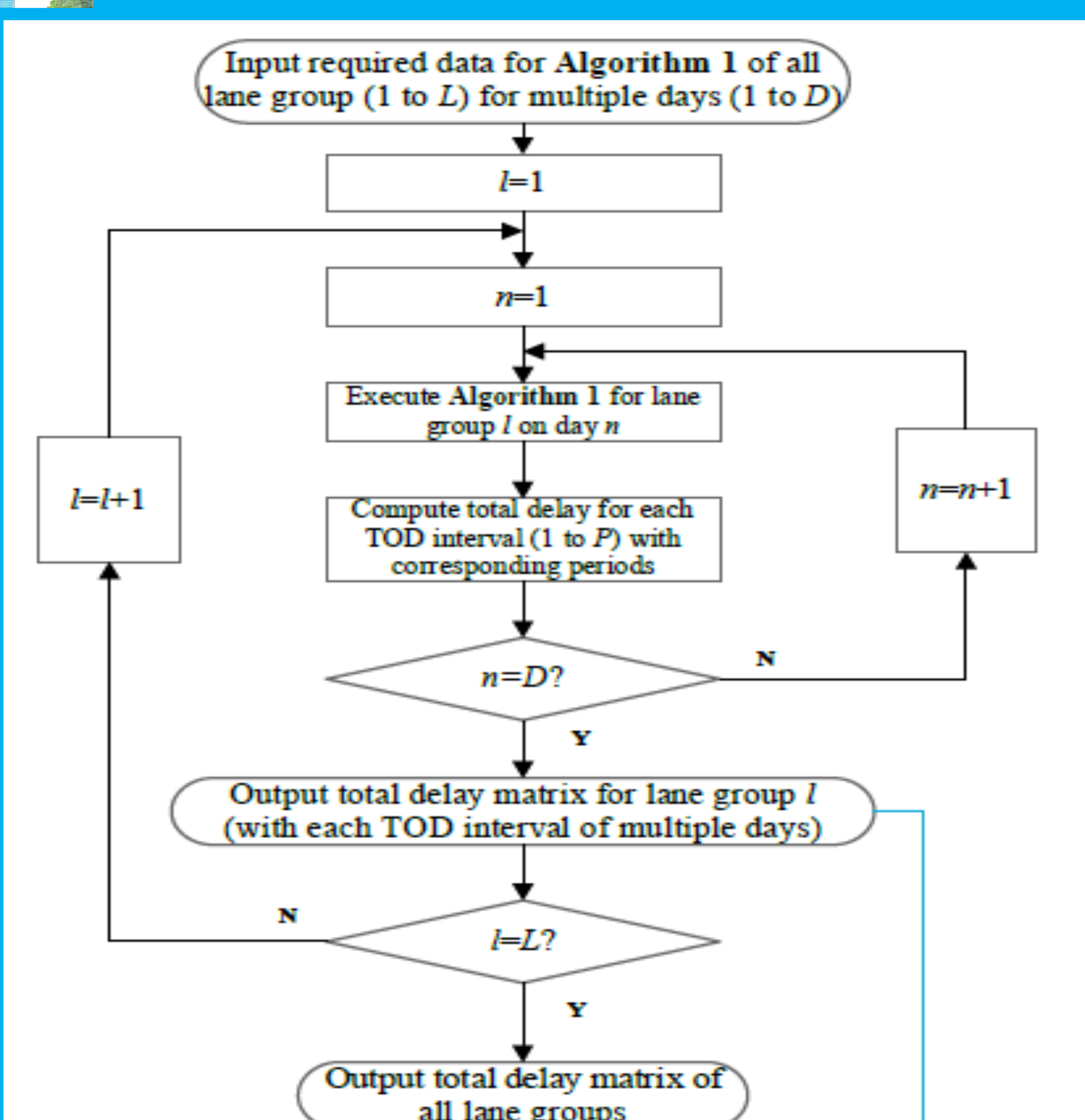
Output: Total delay d_i for each data-collecting interval i.

Initialization: $t_P \leftarrow \sum_P L_P,\; A_2^0 = 0,\; D_2^0 = 0,\; \varepsilon_0 = 0$

for $i=1: I$ **do**
- Update $A_1^i \leftarrow A_2^{i-1}$ and $D_1^i \leftarrow D_2^{i-1}$;
- Update $k_i \leftarrow (A_2^i - A_1^i)/(T_I + \varepsilon_{i-1})$;
- **if** $i > 0$ **and** $i \le t_1$ **then**
  - Update $\varepsilon_i \leftarrow mod(T_I + \varepsilon_{i-1}, C_1)$;
  - **return** d_i with g_1, r_1, C_1, k_1 for $\lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$ cycles;
  - Update $A_2^i \leftarrow A_1^{i-1} + k_i C_1 \lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$;
  - **return** D_2^i for $\lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$ cycles;
- ……
- **else if** $i > t_P$ **and** $i \le t_{P+1}$ $(P + 1 \le \rho)$
  - Update $\varepsilon_i \leftarrow mod(T_I + \varepsilon_{i-1}, C_{P+1})$;
  - **return** d_i with $g_{P+1}, r_{P+1}, C_{P+1}, k_{P+1}$ for $\lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$ cycles;
  - Update $A_2^i \leftarrow A_1^{i-1} + k_i C_{P+1} \lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$;
  - **return** D_2^i for $\lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$ cycles;
- ……
- **endif**

end for

Regulation of Duration of TOD Intervals



Partitions (P) / Days(n)	1	…	P	…	ρ	Daily delay for day n
1	d(1,1)	…	d(1,P)	…	d(1,ρ)	$\sum_{P=1}^{\rho} d(1,P)$
⋮	⋮	…	⋮	…	⋮	⋮
n	d(n,1)	…	d(n,P)	…	d(n,ρ)	$\sum_{P=1}^{\rho} d(n,P)$
⋮	⋮	…	⋮	…	⋮	⋮
D	d(D,1)	…	d(D,P)	…	d(D,ρ)	$\sum_{P=1}^{\rho} d(D,P)$
Partitional delay for multiple days	$\sum_{n=1}^{D} d(n,1)$	…	$\sum_{n=1}^{D} d(n,P)$	…	$\sum_{n=1}^{D} d(n,\rho)$	$\sum_{n=1}^{D}\sum_{P=1}^{\rho} d(n,P)$

Results