Joint Optimization of Time-of-day Intervals and Robust Signal Timing for Isolated Intersection

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ABSTRACT

Time-of-day (TOD) fixed timing is the most common signal control strategy in practice, and it also set a foundation for real-time signal control strategy (e.g., actuated or adaptive) by providing basic background signal plans. Usually, the TOD fixed timing is designed to accommodate structural changes in traffic demand patterns over a typical day. To generate an efficient TOD interval plan with robust signal timings, this paper proposed a bi-level optimization framework in which the duration of each TOD interval and signal timings are the decision variables under the objective of overall efficiency at the upper level and robustness at the lower level. Delay is selected as the performance indicator and calculated based on polygon areas enclosed by accumulative arrival and departure curves. A TOD interval regulation method is also designed to include integer signal cycles to facilitate signal control transition between different TOD intervals. A bilevel evolutionary algorithm based on quadratic approximations (BLEAQ-II) is applied to solve the optimization problem. Finally, a SUMO simulation experiment is conducted to verify the performance of the proposed methodology. The results show the effectiveness and robustness of the proposed method over the existing TOD partition approaches.

Keywords: TOD intervals, Robust signal timing, Bi-level Optimization, Isolated intersection

INTRODUCTION

Time-of-day (TOD) fixed timing, actuated signal control, and adaptive signal control are the three most recognizable signal control strategies in practice (*1*). The traffic community has witnessed the advantages of actuated and adaptive signal control strategies to accommodate traffic fluctuation and their optimality to minimize vehicle delay (*2*). Nevertheless, the TOD fixed timing strategy is still the most widely adopted approach to managing traffic signals for the following reasons: (i) it can balance the effect of traffic demand fluctuation within the day and potentially high equipment and computational budget of other advanced signal control strategies. (ii) it can maintain relatively simple but stable signal operations to account for frequent pedestrian crossing and traffic signal coordination.

Typically, TOD fixed timings will be applied to the same period on multiple days (*3*), and each period is known as the TOD interval. The core challenge in designing a TOD fixed signal timing scheme is to balance the optimality and robustness of signal control. It is not uncommon to see that the fixed timing traffic signal plans are too conservative to cause large waste of green or less robustness or to cause frequent phase failures. In literature, this problem is addressed by properly partitioning the TOD control interval and robustly optimizing signal timing. In general, this is achieved by two independent procedures. The structural traffic demand changes would be first identified based on traffic volume data to partition one day into multiple TOD intervals (*3*), and then the optimized signal timings will be generated for each TOD interval with the objectives such as worst-case delay minimization (*4*). In a real application, the independent two-stage method may face the following problems:

On the one hand, the two-stage method cannot effectively consider the effect of TOD partitions on signal control efficiency. The optimality of a TOD signal control plan is reflected by the total efficiency (e.g., control delay) accumulated over all intervals. Therefore, the TOD partition scheme and the signal timings jointly determine the signal control efficiency (*5*). Thus, the optimal similarity of traffic demand is not equivalent to the optimal TOD signal control efficiency at intersections because of their nonlinear relationship (*6*). More importantly, in the congestion dissipation stage, although the traffic demand has begun to decrease, there may still be residual vehicles that have not been completely discharged. If the TOD control plan is switched at this time, the residual vehicles would not be able to dissipate in time.

On the other hand, the two-stage method is difficult to collectively balance the contradiction between efficiency and robustness. The existing research mostly focuses on the robust optimization of signal timings considering the random characteristics of traffic demand for a given period (*7*). In principle, the enhancement of signal timing robustness comes at the expense of control efficiency. Therefore, the rationality of TOD interval partition determines the difficulty of balancing efficiency and robustness in the process of signal timing optimization, that is, under the premise of ensuring robustness, less efficiency would be sacrificed. In this context, TOD interval partitions also need to be collaboratively considered when determining the robust signal timing.

To overcome the shortcomings brought by the two-stage method, this paper proposes a bi-level optimization framework that can integrally consider the efficiency and robustness, and jointly optimize both TOD interval partitions and signal timings.

Literature Review

Most of the literature since the 2000s separately focuses on methods for two respective stages (i.e., TOD interval partitions and robust signal timings). Few studies have jointly optimized the TOD interval partitions and the robust signal timings considering multiple demand scenarios across multiple days.

TOD signal control interval partitions

A large number of earlier studies have treated the TOD interval partition problem as a cluster problem using traffic data such as volume, speed, occupancy, etc. Generally, both hierarchical and non-hierarchical clustering techniques were investigated (*8*-*9*). With non-hierarchical clustering algorithms, the *k*-means method was first proposed by Wang et al. (*10*) in partitioning TOD intervals, in which smoothing techniques are needed for noisy points. To smooth the small cluster generated by the traditional *k*-means which causes discontinuous TOD interval, Guo & Zhang (*11*) further considered the time dimension in *k*-means clustering. Dong et al. (*12*) combined the Isomap algorithm with the *k*-means algorithm to guarantee reasonable interval length. Song et al. (*13*) proposed several adjustment methods for the *k*-means algorithm to avoid non-contiguous periods. Wan et al. (*14*) applied the bisecting *k*-means algorithm to determine TOD breakpoints based on the variation of queuing shockwave speeds.

With hierarchical clustering algorithms (*15*), Smith et al. (*3*) applied the CART decision tree for classifying volume and occupancy data. Jun & Yang (*16*) clustered the input data by forming up the Kohonen Neural Network and applied the determined TOD breakpoints on a corridor. Shen et al. (*17*) developed a fast and robust fuzzy C-Means clustering approach to obtain TOD breakpoints.

Different from the clustering techniques, some studies formulate the TOD interval partition problem as an optimization problem, which can be solved by heuristic algorithms such as genetic algorithms (GA) (*18*). More recently, Coogan et al. (*19*) established a convex optimization model that considered time segment cost and adjusted TOD breakpoints through partial least squares predictions. A dynamic programming formulation was proposed in (*20*) which was solved by a recursive algorithm to optimize TOD breakpoints.

For both clustering and optimizing techniques, the TOD interval partitions aim to discover the structural patterns in the input traffic data without explicitly considering its impacts on signal control efficiency. Additional methods are required for jointly considering the TOD interval partitions and signal timings.

Robust Optimization of Traffic Signal Timing

To deal with the traffic demand uncertainty, robust optimization has recently been paid more attention to. It uses the uncertain set (e.g., day-to-day hourly flow) to describe the traffic uncertainty. Also, the worst scenario will be minimized to find the optimal solution. Specifically, it is since the work of Yin (*21*) that the study of robust optimal signal timing has gained momentum, in which three robust models were proposed to obtain robust optimal signal timing. Most subsequent studies have developed new methods based on one of these three robust optimization models as summarized below.

The first model is the scenario-based mean-variance (MSD) optimization. It uses a weighted linear combination of the mean and standard deviation (SD) of traffic performance measures (e.g., delay) as the objective function. Zheng et al. (*22*) formulated a reliability-based signal optimization problem based on an analytic model of travel time distribution, in which the model assumes periodic average demand. Chen et al. (*7*) proposed a simulation-based optimization framework for reliable large-scale signal control, with consideration of mean and SD of travel time. Chen et al. (*23*) generated a robust operational scheme with minimized mean and SD of vehicular delay of signalized intersections with contraflow left-turn lanes.

The second model is the scenario-based conditional value-at-risk (CVaR) minimization, which uses the CVaR value to represent robustness. Zhang et al. (*24*) implemented the cell-transmission model (CTM) and minimized the high-consequence mean excess delay as CVaR by optimizing green splits, cycle length, and offsets along an arterial. Based on (*24*), Zhang et al. (*25*) extended the CTM to define mean excess exposure as the environmental risk. Papatzikou and Stathopoulos (*26*) minimized the excess delay calculated from the Highway Capacity Manual (HCM) as the risk of being oversaturated (CVaR) to improve signal control performance.

Different from MSD and CVaR, the min-max optimization model (MNMX) is suitable for circumstances where traffic data are limited. The MNMX is to minimize the potential maximum delay within the likelihood region. Li (*27*) formulated a binary integer programming model for robust signal timing by minimizing maximum delays. Tettamanti et al. (*28*) presented a model predictive control approach for minimizing the objective function with the appearance of the worst case. Yang et al. (*4*) used demand intervals (similar to the TOD intervals) to represent time-dependent demand fluctuation as a set of uncertain variables for adjacent time horizons. Chiou (*29*) proposed a bi-level robust model solved by trust-region cutting plane projection considering travel cost and capacity expansion. A min-max mathematical program with the equilibrium constraints model was then established by Chiou (*30*) to minimize the maximum delays and number of stops considering uncertain Origin-Destination demand.

More recently, Shirke et al. (*31*) proposed a meta-heuristic approach and compared it with the above three robust optimization models. Although some attempt has focused on developing new methods for more robust signal timings, the three above-mentioned models are still active for robust optimization. However, most of the literature only considers a predefined period. In practice, the overall robustness for multiple TOD intervals needs to be considered as well.

Integrated Optimization of TOD breakpoints and signal timings

Attempts have merged in determining TOD interval partitions and signal timings within an integrated framework. Wong & Woon (*32*) proposed an iterative method that can re-assign the TOD breakpoints based on the optimized signal plan. Xu et al. (*33*) put the *k* central point clustering algorithm into a two-order optimization framework and used a piecewise point division model. Abbas and Sharma (*34*) proposed a method based on the non-dominated sorting genetic algorithm for optimizing delay, stops and degree of detachment with signal timing generated by PASSER V. Park and Lee (*35*) conducted the greedy search algorithm based on simulation results to finalize TOD breakpoints. Lee et al. (*36*) considered the TOD control transition cost and developed a GA-based optimizer for optimal breakpoints. Zheng et al. (*37*) proposed a simple delay model based on the HCM formula to determine the TOD breakpoints. García-Ródenas et al. (*5*) formulated a bilevel TOD determination problem solved by memetic algorithms with four approaches to determine the optimal number of TOD intervals. Most studies solve the integrated optimization problem in a deterministic way. To the best of our knowledge, a well-balancing framework between TOD interval partitions and robust signal timing has not been identified.

Contribution

This paper fills the research gap for jointly optimizing the TOD interval partition and signal timing under the uncertainty of traffic demand, and it has the following contributions: (i) An integrated bi-level optimization framework is established with delay minimization as the upper level (UL) and the robustness as the lower level (LL). (ii) The relationship between the duration of TOD interval and the signal cycles is explicitly discussed for smoothing the signal control transition. (iii) Scenario-based method is adopted to quantify traffic demand uncertainty and calculate vehicle delay over multiple TOD intervals given historical traffic data.

Paper Structure

The rest of this paper is organized as follows: Section 2 formulates a bi-level optimization framework for the TOD interval partition problem with robust signal timings which is solved by the BLEAQ-II algorithm. The delay calculation method and the TOD interval regulation method are also discussed in Section 2. Section 3 conducts a case study based on the SUMO simulation platform, in which the proposed method is compared with the benchmark methods. Section 4 gives the concluding remarks and future works.

METHODOLOGY

Notations and Assumptions

To formulate the TOD interval partition problem, we first introduce the following notations in **TABLE 1** for the optimization framework proposed in the next section:

In this paper, we assume that: (i) the number of TOD intervals is pre-determined; (ii) advance detectors are installed on each approach of the intersection, which would provide accurate accumulative arrival curves (CAC) with a certain data collection interval (e.g., 5-min) for each lane group; (iii) the saturation flow rate and signal lost time are constant while traffic demand can fluctuate day-by-day.

TABLE 1 Notations

Td	total delay for all TOD intervals on *D* days (s)
l	lane group (*L* lane groups in total)
l^c	set of critical lane groups
n	day (*D* days in total)
P	TOD interval (ρ TOD intervals in total)
d_P^n	total delay of TOD interval *P* on day *n* (s)
$\overline{d_P^n}$	average total delay of TOD interval *P* for *D* days (s)
$\sigma_{d_P^n}$	SD of total delay of TOD interval *P* for *D* days (s)
g_P^l	effective green time of lane group *l* in TOD interval *P* (s)
C_P	cycle length in TOD interval *P* (s)
L_P	duration of TOD interval *P* (s)
TL	sum of the durations of all TOD intervals (s)
L_t	total lost time for each cycle (s)
$L_P^{Min}(L_P^{Max})$	minimum (maximum) duration of TOD interval (s)
$g_P^{Min}(g_P^{Max})$	minimum (maximum) effective green time (s)
$C_P^{Min}(C_P^{Max})$	minimum (maximum) cycle length (s)
γ	robustness ratio

Bi-level Optimization Framework

The TOD interval partition with robust signal timings problem can be modeled as a bi-level optimization framework. For a more detailed discussion about bi-level optimization see (*38*).

In the proposed bi-level optimization framework, we consider the total delay minimization as the UL objective while the robust signal control is the LL objective. In other words, the TOD interval partition problem will be considered under the robust signal timing scheme. Mathematically, the bi-level optimization framework can be written as follows.

$$\min_{g_P^l, C_P, L_P} Td = \sum_{n=1}^{D} \sum_{l=1}^{L} \sum_{P=1}^{\rho} d_P^n\left(g_P^l, C_P, L_P\right) \tag{1}$$

$$s.t.(UL)$$

$$\sum_{P=1}^{\rho} L_P = TL \tag{2}$$

$$L_P^{Min} \leq L_P \leq L_P^{Max} \tag{3}$$

$$mod(L_P, C_P) = 0 \tag{4}$$

$$\min_{g_P^l, C_P} \sum_{l=1}^{L} \sum_{P=1}^{\rho} \overline{d_P^n}\left(g_P^l, C_P\right) + \gamma \sigma_{d_P^n}\left(g_P^l, C_P\right) \tag{5}$$

$$s.t.(LL) \sum_{l \in l^c} g_P^l + L_t = C_P \tag{6}$$

$$g_P^{Min} \leq g_P \leq g_P^{Max} \tag{7}$$

$$C_P^{Min} \leq C_P \leq C_P^{Max} \tag{8}$$

Equation 1 and **Equation 5** are UL and LL objective functions, respectively. First, we keep L_P constant and solve the LL problem to obtain the optimal g_P^l and C_P, and then substitute the optimal g_P^l and C_P into the UL problem to solve the optimal L_P. We selects the MSD optimization model to obtain robust signal timings. The total delay (*Td*) and average and SD of total delay ($\overline{d_P^n}$ and $\sigma_{d_P^n}$) would be calculated according to models described in Section 2.2.1. For UL constraints, **Equation 2** means the sum of each TOD interval duration equals a constant value (e.g., 86400s for one-day time), and **Equation 3** considers the minimum and the maximum interval duration to avoid high transition costs. And the constraint on the divisibility relationship between L_P and C_P (**Equation 4**) will be further discussed in Section 2.2.2 to smooth the signal transition process and guarantee TOD control continuity. In addition, linear constraints between effective green time and the cycle time are listed from **Equation 6** to **Equation 8**.

Delay Estimation Method

In this paper, the traffic demand and supply patterns are captured by the cumulative arrival and departure curves (CAC and CDC), respectively. An example of CAC and CDC for a given lane group within a signal cycle is shown in **Figure 1**. The orange and light blue curves are CAC and CDC, respectively, in which *g* is the green time duration and *C* is the cycle length. In practice, the approximate CAC (the dark blue curves in **Figure 1**) can also be generated as a segmented polyline (segmental linearization), and the CDC is also a segmented polyline once the signal timings and saturation flow rates are determined. Compared with the traditional delay model such as Webster's uniform delay model (*39*), the arrival-departure curve can better handle the oversaturated condition.

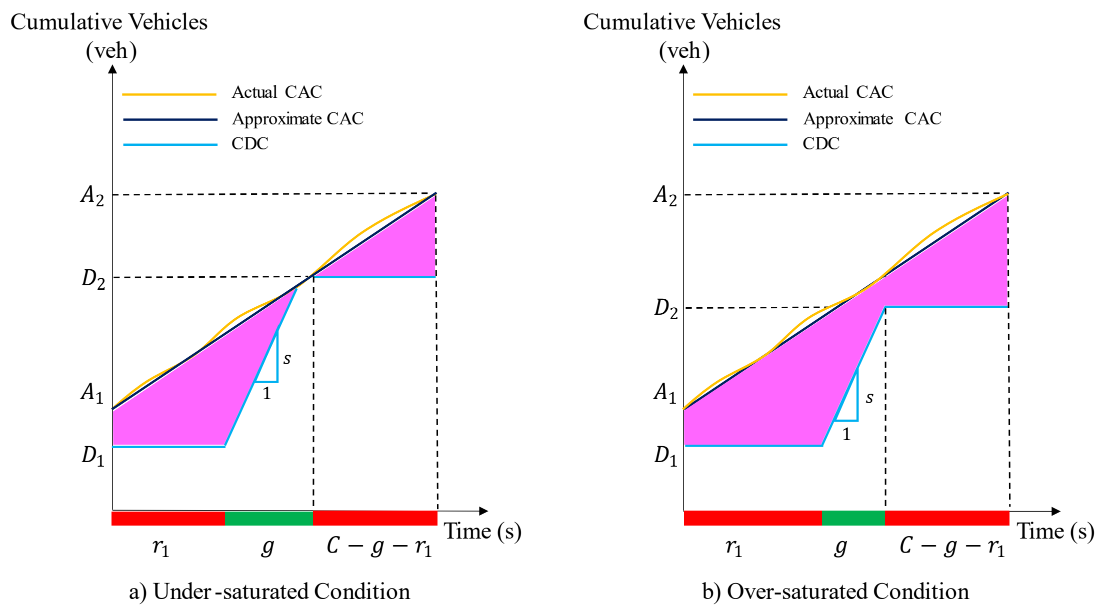


Figure 1 Cumulative arrival and departure curve

As shown in Figure 1, the slope of CAC is the arrival flow rate which varies among different time intervals, and the slope of the CDC represents the capacity of the signalized lane group, which remains as 0 if it is a red signal and turns to the saturation flow rate *s* before intersecting with the CAC. Note that the CDC will coincide with the CAC after intersecting in the green phase as shown in **Figure 1a**. For a given signalized intersection, assuming we have *m* lane groups and *n* analyzed days, we will generate *m*×*n* CACs and CDCs.

For a given TOD interval from the UL, we calculate the interval total delay through a cycle-by-cycle approach using the CDC and CAC. According to HCM, the average control delay is the summation of the uniform delay (UD), the incremental delay (ID), and the initial queue delay (IQD). We first discuss the UD and IQD from the cyclic perspective and then sum multiple cycles' UD and IQD up to obtain the UD and IQD of a given period.

For each cycle, the sum of total UD and IQD can be calculated as the area enclosed by the CAC and CDC. To simplify the calculation and improve the computational efficiency, we assume that vehicles arrive at the intersection at a uniform arrival rate of *k* during each cycle (the slope *k* of the approximate CAC in **Figure 1**) within the given period.

As a section of the daily CAC and CDC, the CAC and CDC within one signal cycle (**Figure 1**) rely on the CAC and CDC from the last cycle. So, the initial queue for the analysis cycle, $A_1 - D_1$, is determined by the last cycle, and the final cumulative arrivals of the analysis cycle (A_2) equal to $A_1 + kC$. Here, we generalize the typical triangular area to the polygon area (pink area in **Figure 1**) to handle the initial queue problem by A_1 and D_1 and capture the TOD control transition process by r_1.

The area calculation approach can be categorized into two cases according to conditions whether the analysis cycle is under-saturated or over-saturated (whether the CDC can intersect with the CAC within the analysis cycle or not). Both conditions depend on the area of the trapezoid enclosed by CAC and the x-axis minus the area of the polygon enclosed by CDC and the x-axis. Hence, we can obtain the cyclic total UD and IQD for under-saturated and over-saturated conditions by **Equation 9**. After the cyclic analysis is completed, we update the final cumulative departures D_2 to determine the next cycle's D_1 by **Equation 10**.

$$d_c = \begin{cases} C \times \left(A_1 + \frac{1}{2}kC\right) - \frac{g}{2}us - \left(C - r_1 - \frac{u}{2} - \frac{g}{2}\right) \times (kr_1 + kg + A_1 - D_1) - D_1 C & 0 \le u \le g \\ C \times \left(A_1 + \frac{1}{2}kC\right) - gs\left(C - r_1 - \frac{g}{2}\right) - D_1 C \quad else & (u = \dfrac{kr_1 + A_1 - D_1}{s-k}) \end{cases} \tag{9}$$

$$D_2 = \begin{cases} A_1 + k(g + r_1) & 0 \le u \le g \\ A_1 + gs \qquad else & (u = \dfrac{kr_1 + A_1 - D_1}{s-k}) \end{cases} \tag{10}$$

Here, we use the HCM formula as shown in **Equation 11** to calculate the average ID of a given period *T* which contains multiple signal cycles.

$$d_I = 900T\left[\left(\frac{kC}{gs} - 1\right) + \sqrt{\left(\frac{kC}{gs} - 1\right)^2 + \frac{4C^2k}{g^2s^2T}}\right] \tag{11}$$

Then, for a given analysis period containing *B* cycles with a uniform arrival rate *k* (*T*=*BC*), we can obtain the periodic total control delay by multiplying the average control delay with the traffic volume (*kBC*) in **Equation 12**. The sum of the total UD and IQD is the first item, and the total ID is the second item.

$$\{d_P\}_B = \{d_C\}_B + \{d_I\}_B = \sum_{b=1}^{B}\{d_C\}_B + 900kB^2C^2\left[\left(\frac{kC}{gs} - 1\right) + \sqrt{\left(\frac{kC}{gs} - 1\right)^2 + \frac{4Ck}{g^2s^2B}}\right] \tag{12}$$

By applying the modified delay estimation process based on HCM, we can more intricately capture the total delay for a shorter time interval. Besides, the update mechanism of CACs and CDCs brings continuity in estimating IQD, especially for the oversaturated intersections.

Regulation of the duration of TOD Intervals

In this paper, the durations of the TOD intervals are the decision variables. For an intersection that has *P* TOD intervals, it has *P* cycle lengths, *P* signal timing schemes, and *P*-1 TOD breakpoints. Each TOD interval contains several data-collecting periods (e.g., 5-min) and the total delay of each period can be calculated by the above-mentioned cycle-by-cycle process. In most cases, the duration of the data-collecting period or TOD interval does not happen to be equal to the integer cycle length. So, we propose an adjustment method to guarantee the TOD intervals as the integer numbers of the data-collecting period and the cycle length, so that the convenience of the signal control transition process can be achieved.

TABLE 2 Pre-determined Parameters

Algorithm 1: Method of adjusting TOD intervals for estimating delays

Input: Total No. of data-collecting periods *I* (with each duration of T_I), duration set of TOD intervals $\{L_P\}_{P=1}^{\rho}$, cumulative arrivals A_2, effective green time set $\{g_P\}_{P=1}^{\rho}$, effective red time before green set $\{r_P\}_{P=1}^{\rho}$, and cycle length set $\{C_P\}_{P=1}^{\rho}$.

Output: Total delay d_i for each data-collecting interval *i*.

Initialization: $t_P \leftarrow \sum_P L_P$, $A_2^0 = 0$, $D_2^0 = 0$, $\varepsilon_0 = 0$

for *i=1: I* **do**

  Update $A_1^i \leftarrow A_2^{i-1}$ and $D_1^i \leftarrow D_2^{i-1}$;

  Update $k_i \leftarrow (A_2^i - A_1^i)/(T_I + \varepsilon_{i-1})$;

    if $i > 0$ **and** $i \le t_1$ **then**

      Update $\varepsilon_i \leftarrow mod(T_I + \varepsilon_{i-1}, C_1)$;

    return d_i via **Equation 12** with g_1, r_1, C_1, k_1 for $\lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$ cycles;

    Update $A_2^i \leftarrow A_1^{i-1} + k_i C_1 \lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$;

    return D_2^i via **Equation 10** for $\lfloor (T_I + \varepsilon_{i-1})/C_1 \rfloor$ cycles;

  ……

  else if $i > t_P$ **and** $i \le t_{P+1}$ $(P + 1 \le \rho)$

      Update $\varepsilon_i \leftarrow mod(T_I + \varepsilon_{i-1}, C_{P+1})$;

    return d_i via **Equation 12** with $g_{P+1}, r_{P+1}, C_{P+1}, k_{P+1}$ for $\lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$ cycles;

    Update $A_2^i \leftarrow A_1^{i-1} + k_i C_{P+1} \lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$;

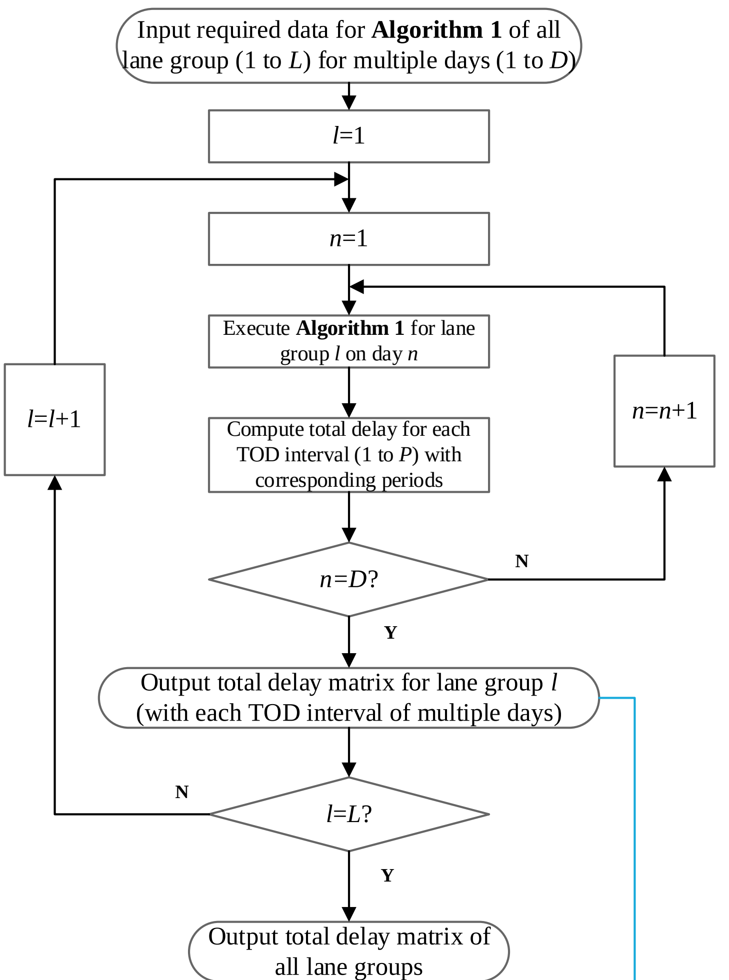
    return D_2^i via **Equation 10** for $\lfloor (T_I + \varepsilon_{i-1})/C_{P+1} \rfloor$ cycles;

  ……

  endif

end for

For a data-collecting period, we divide the period by cycle length, the quotient obtained is the number of complete cycles contained in the period, and the remainder is called the residual time. Next, we consider the next period as the sum of the period and the residual time from the last period, and we will obtain the quotient and the reminder as well. Then, repeat the above process till the last data-collecting period of the analyzed TOD interval, and it will also generate a residual time, and we will continuously transfer it to the next period and set the TOD breakpoint as the end of the integer cycle rather than the end of the last data-collecting period. For example, for a TOD interval containing 24 5-min data-collecting periods with no residual time from the last interval (24×300=7200s), if the cycle length is 60s, then it contains exactly 5 cycles. But if the cycle is 70s, it contains 4 complete cycles and generates 20s of residual time for the first period. We transfer the 20s to the next 5-min period which makes the next period 320s, and it contains 4 complete cycles and generates 40s of residual time. In this way, when it comes to the last 5-min period, it will generate 60s of residual time (7200=102×70+60), and we will transfer the 60s to the next data-collecting period for the next TOD interval and set the breakpoint at 7140s. The pseudo-code of the above process is listed in **TABLE 2** for one day of a single lane group. Note that we assume no residual time from the last day for each day's first TOD interval.



Days(n) \ Partitions (P)	1	…	P	…	ρ	Daily delay for day n
1	d(1,1)	…	d(1,P)	…	d(1,ρ)	$\sum_{P=1}^{\rho} d(1,P)$
⋮	⋮	…	⋮	…	⋮	⋮
n	d(n,1)	…	d(n,P)	…	d(n,ρ)	$\sum_{P=1}^{\rho} d(n,P)$
⋮	⋮	…	⋮	…	⋮	⋮
D	d(D,1)	…	d(D,P)	…	d(D,ρ)	$\sum_{P=1}^{\rho} d(D,P)$
Partitional delay for multiple days	$\sum_{n=1}^{D} d(n,1)$	…	$\sum_{n=1}^{D} d(n,P)$	…	$\sum_{n=1}^{D} d(n,\rho)$	$\sum_{n=1}^{D}\sum_{P=1}^{\rho} d(n,P)$

Figure 2 Flowchart for total delay calculations over multiple lane groups and multiple days

Also, the CAC generated in **Algorithm 1** can be regarded as the reasonable approximation of the original CAC if T_I is relatively small (i.e., 5-min or 10-min) and the generated CDC can well handle different *g*, *C*, and r_1 for different TOD intervals. And to obtain the delays required in the bi-level optimization framework, we repeat **Algorithm 1** for multiple days of all lane groups. As shown in the flowchart in **Figure 2**, for each TOD interval, the total delay matrix is generated which the value in the lower right corner of the table is for UL function and the partitional total delays in each column for multiple days are for average and SD of total delay in the LL objective function.

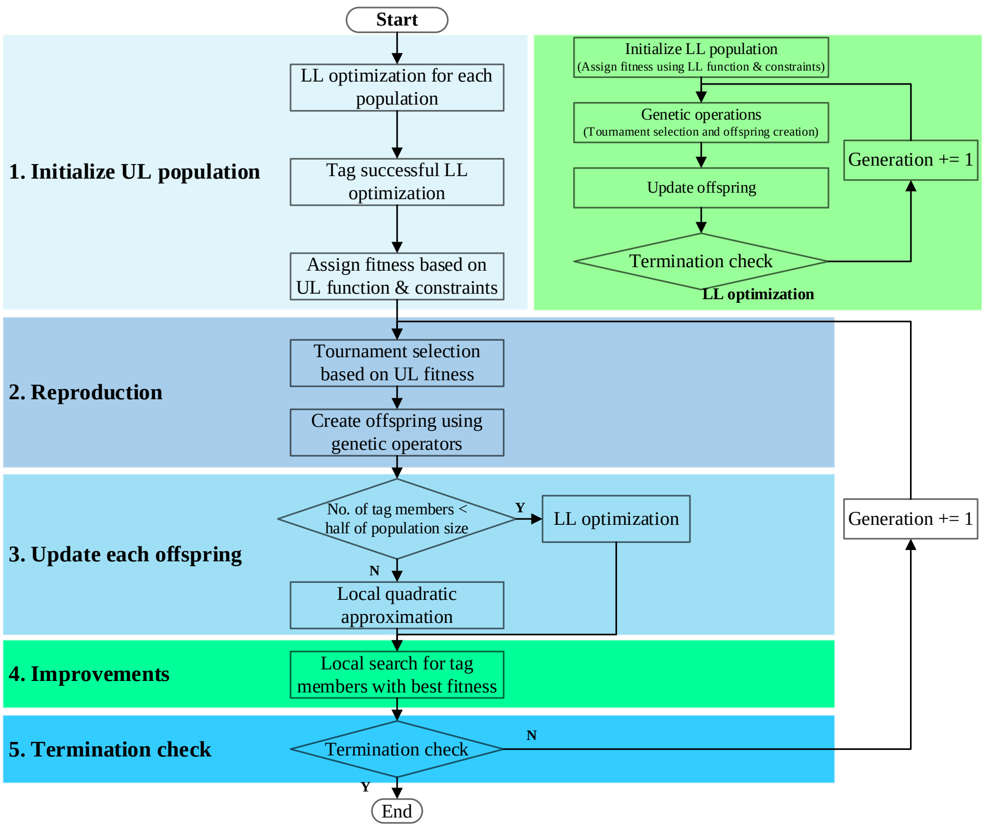
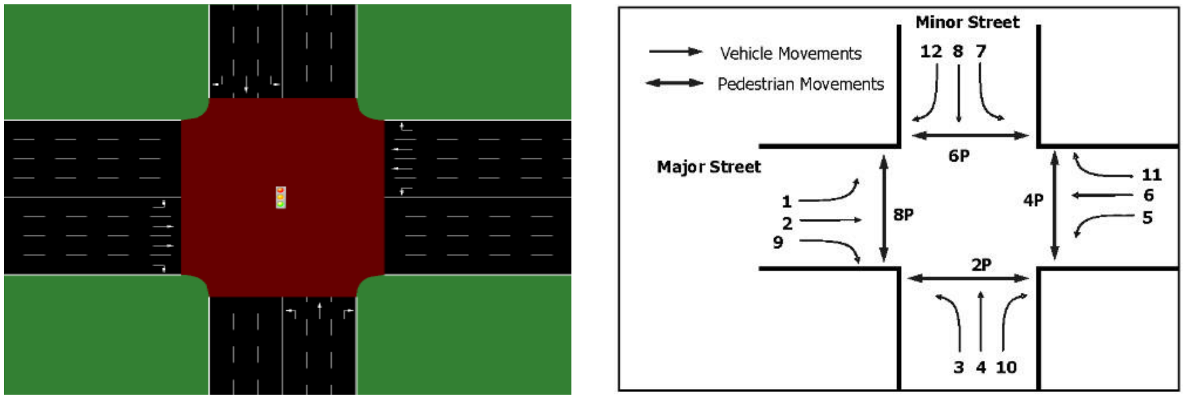


Figure 3 BLEAQ-II algorithm

Optimization Algorithm

From the formulation of the bilevel optimization framework, the TOD interval partition with robust signal timings problem can be considered an NP-hard problem. Hence, we select an improved version of the bilevel evolutionary algorithm based on quadratic approximations (BLEAQ-II) to solve the problem in this case. Compared with the traditional BLEAQ algorithm, BLEAQ-II has fewer overall function evaluations. **Figure 3** shows the step-by-step computation procedure for the BLEAQ-II algorithm. More details about BLEAQ-II are introduced in (*40*).

For the proposed bi-level optimization problem, we first conduct the LL optimization to obtain the robust signal timings, then execute steps 1 to 5 in **Figure 3** to solve the UL optimization problem with the optimized LL results. After the iteration process is completed, we finalize the results by outputting the optimal TOD partitions (from UL populations) and optimal signal timings (from LL populations).



a) Experiment Intersection in SUMO

b) Traffic Flow Movements

Figure 4 Experiment setup

CASE STUDY

Experiment setup

To simulate the real-world environment, we built a simulation platform through Simulation of Urban MObility (SUMO) software (version 1.10.0) run by a desktop computer with an Intel Core i5-7200U 2.7 GHz CPU and a memory of 16 GB (the embedded python version is 3.8). The relative parameters of the simulation are listed in **TABLE 3**. As shown in **Figure 4**, a typical 4-leg signalized intersection was constructed. To create traffic demand fluctuation within the day, we defined the mean flow rate of each movement for every hour under the fluctuation, and vehicles arrive in each hour follow the Poisson distribution with the mean flow rate plus a random integer to account for a moderate traffic demand variations over multiple days. Then, we collected the flow data based on 5-min periods within 24 hours (288 5-min periods for a day). As shown in **Figure 5**, morning peak and evening peak (as oversaturated conditions) are generated for all movements, in which movements 1, 2, 3, and 8 are considered as critical movements. As for the right-turn movements (9-12), the 5-min right-turn flow within the day is generated as a constant volume plus a random integer.

TABLE 3 Pre-determined Parameters in SUMO

Parameters	**Value**
Vehicle in SUMO	
max. acceleration (m/s^2)	0.8
max. deceleration (m/s^2)	4.5
max. speed (m/s)	16.67
min. gap (m)	2.5
length (m)	5
drivers' imperfection factor	0.5
lost time (s/phase)	4
Lane in SUMO	
max. speed (m/s)	13.89
width (m)	3.5
yellow time (s/phase)	3

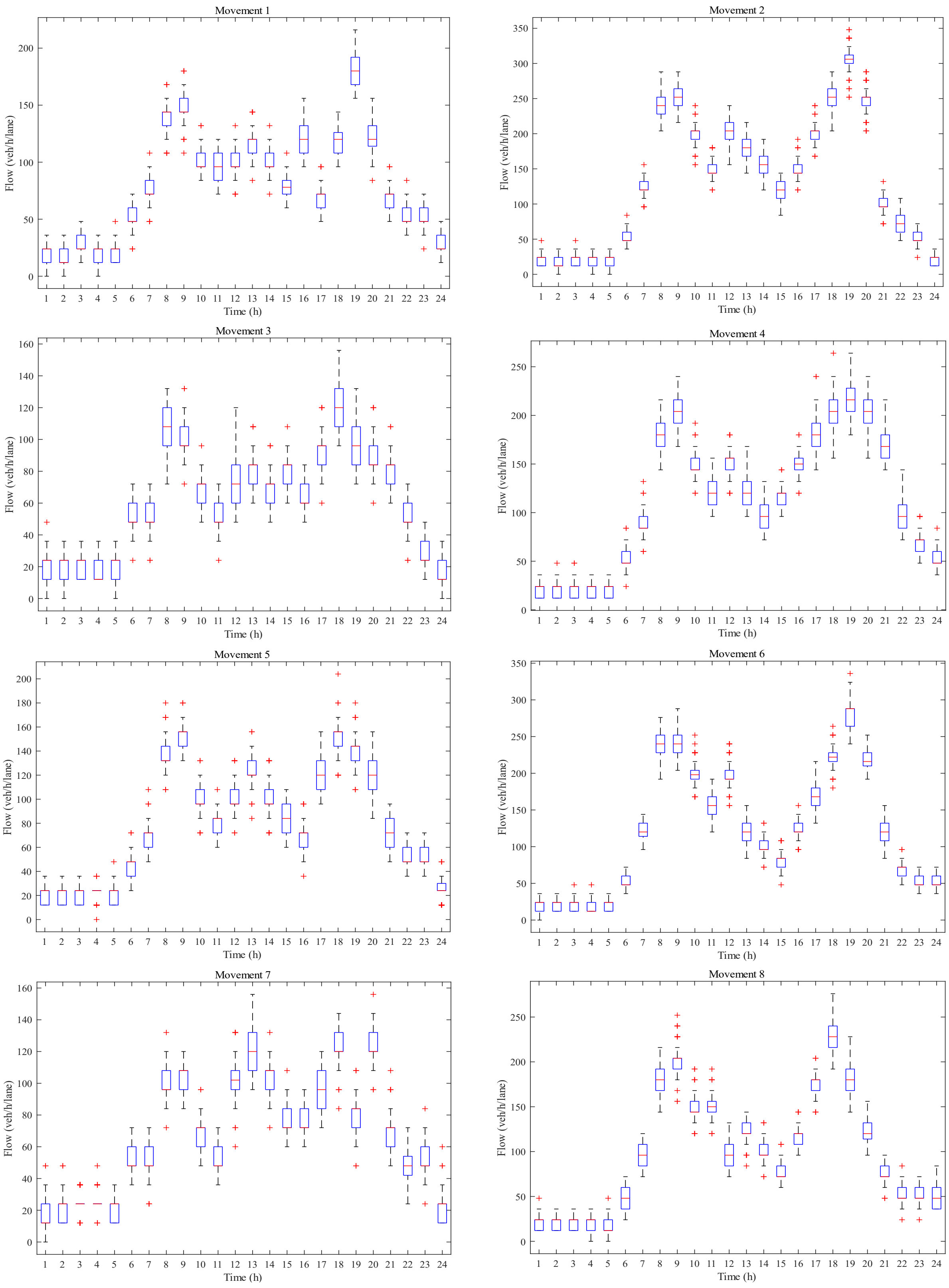


Figure 5 Movement demand fluctuation

We generate seven days' scenarios as the training set and ten days' scenarios as the test set. On this basis, we collect 56 cumulative arrival curves for the training set. For vehicle types of the simulation, we assume that all vehicles are passenger vehicles and we use the default vehicle type provided by a SUMO example.

To validate the effectiveness of the proposed methodology, we use two signal control benchmarks: (i) Single plan traffic signal control scheme (only one signal plan throughout the day), which conducts the HCM method based on mean flow for 7 days (Scheme 1). (ii) TOD signal control scheme generated through *k*-means clustering algorithm (also known as two-stage approach), with five TOD intervals based on the fluctuated pattern of the traffic flow (Scheme 2). For Scheme 2, we used the mean flow of 7 days for all 5-min periods of all lane groups and input them into the *k*-means clustering algorithm embedded in MATLAB R2022a, and the results of clustering are shown in **Figure 6**. Then, we applied the HCM method for each TOD interval to obtain the benchmark signal timings.

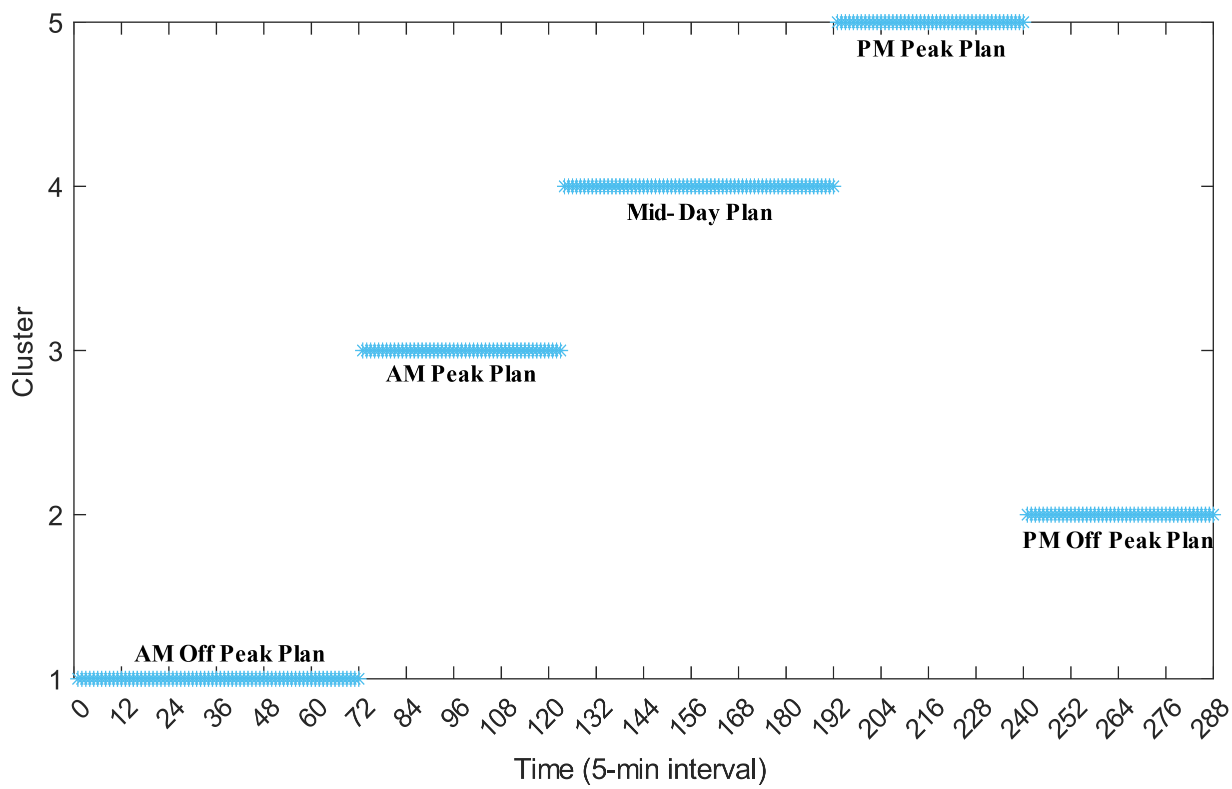


Figure 6 *K*-means clustering results for TOD benchmark plan

Optimization Results

We implemented the BLEAQ-II algorithm by MATLAB R2022a on the same computer that operated the SUMO simulation platform. Firstly, we encode the durations of five TOD intervals as four UL decision variables and 20 signal timings (4 phases for each TOD interval) as 20 LL decision variables. Bounds and constraints on TOD interval durations and signal timings are listed in **TABLE 4** accompanied by other pre-determined BLEAQ-II parameters. The robustness ratio is set as 2.1 according to (*41*). An experiment was conducted in which the BLEAQ-II was converged at the generation of 574 as shown in **Figure 7**. Note that we use the negative form for UL and LL function for the maximum objective required in BLEAQ-II.

TABLE 4 Pre-determined Parameters for BLEAQ-II

Parameters	**Value**
population size	50 for UL and 100 for LL
dimensions of decision variables	4 for UL and 20 for LL
maximum generations	1500 for UL and LL
minimum TOD interval duration	12 5-min (1 h)
maximum TOD interval duration	120 5-min (10 h)
minimum effective green time	10 s
maximum effective green time	100s
maximum cycle length	180s
stopping criteria	1e-05 for UL and LL
crossover probability	0.9 for UL and LL
mutation probability	0.25 for UL and 0.05 for LL
saturation flow rate (veh/h)	900
robustness ratio	2.1

After the converged optimization results were obtained (denoted as Scheme 3), we compared the optimal function value with that of the TOD benchmark plan. For the UL function value, the total delay value of the benchmark scheme is -1.1777e+09 and is -5.3050e+08 for the optimal scheme. For LL function value, the benchmark mean-variance result is -3.1422e+08 while -1.3939e+08 for the optimal value, which shows effective optimization performance.

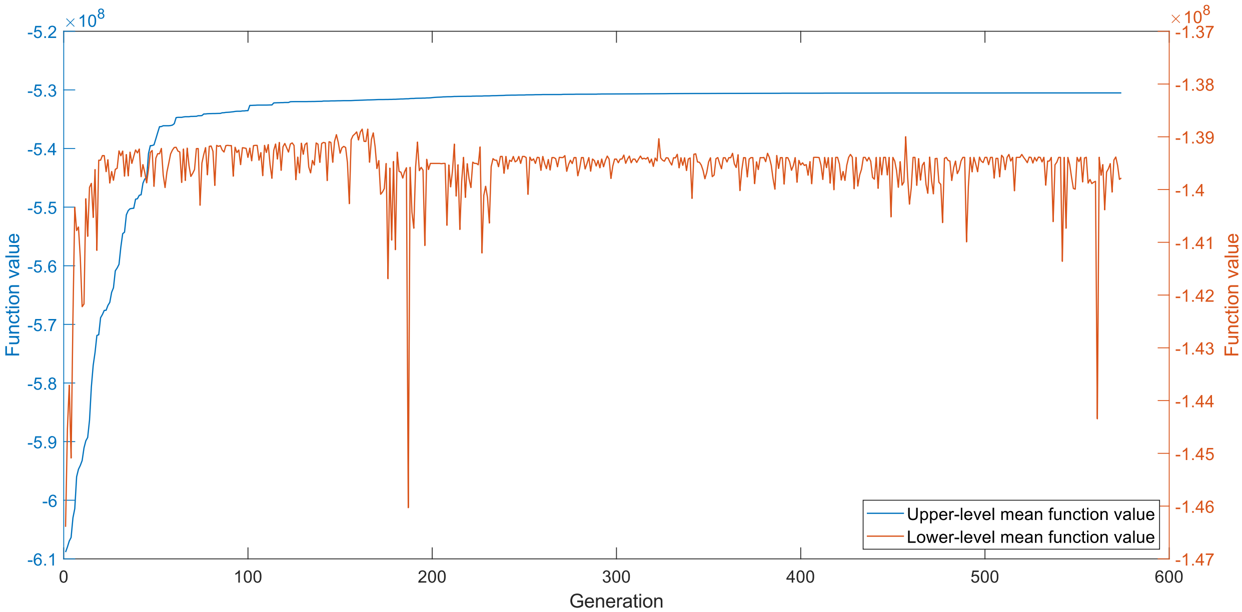
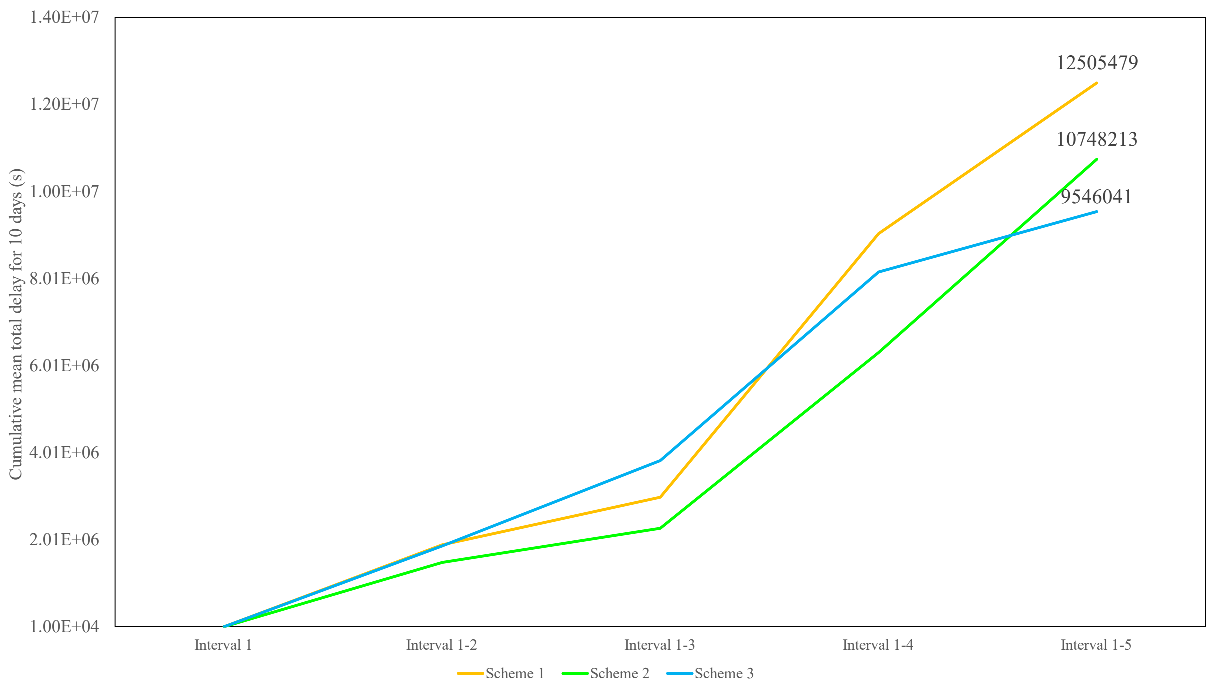


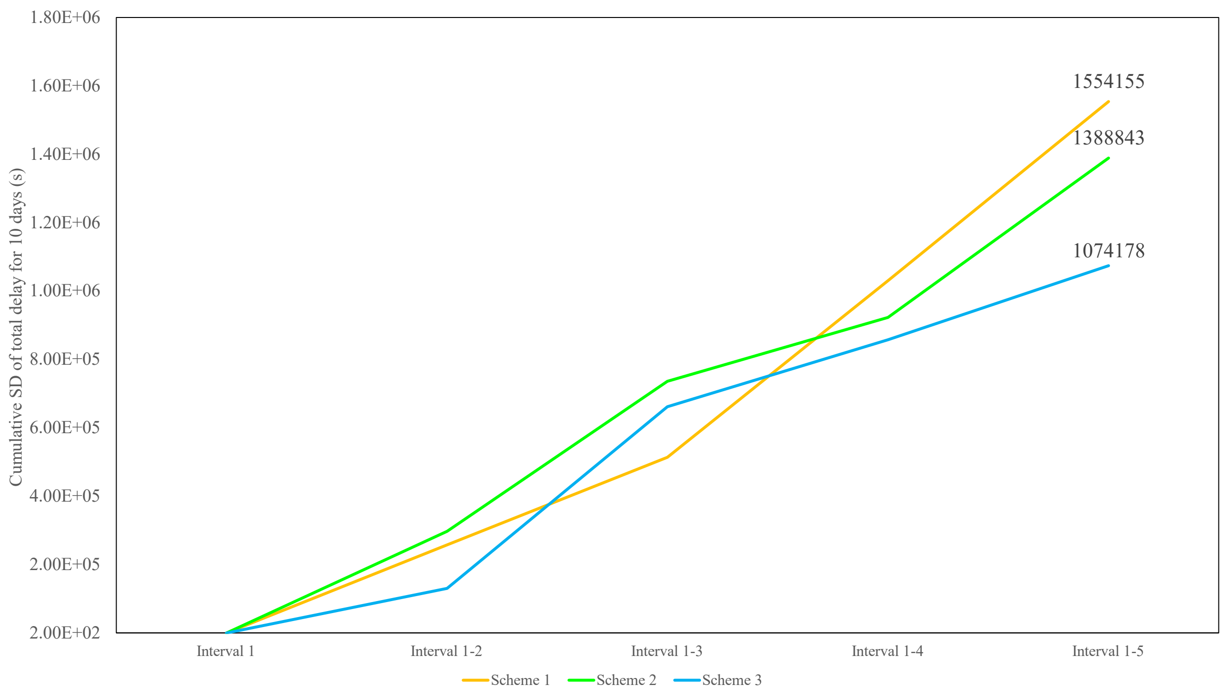
Figure 7 Optimization performance of BLEAQ-II for the robust TOD problem

Performance Evaluation

Next, we input three schemes into the SUMO simulation platform to validate the proposed methodology. It has been pre-determined that Scheme 1 has only one signal phase sequence while Scheme 2 and 3 have five sequences for corresponding five TOD intervals. We first compare the results of three schemes from the partitional perspective.



a) Mean of partitional total delay of the test set



b) SD of partitional total delay of the test set

Figure 8 Comparison of partitional total delay

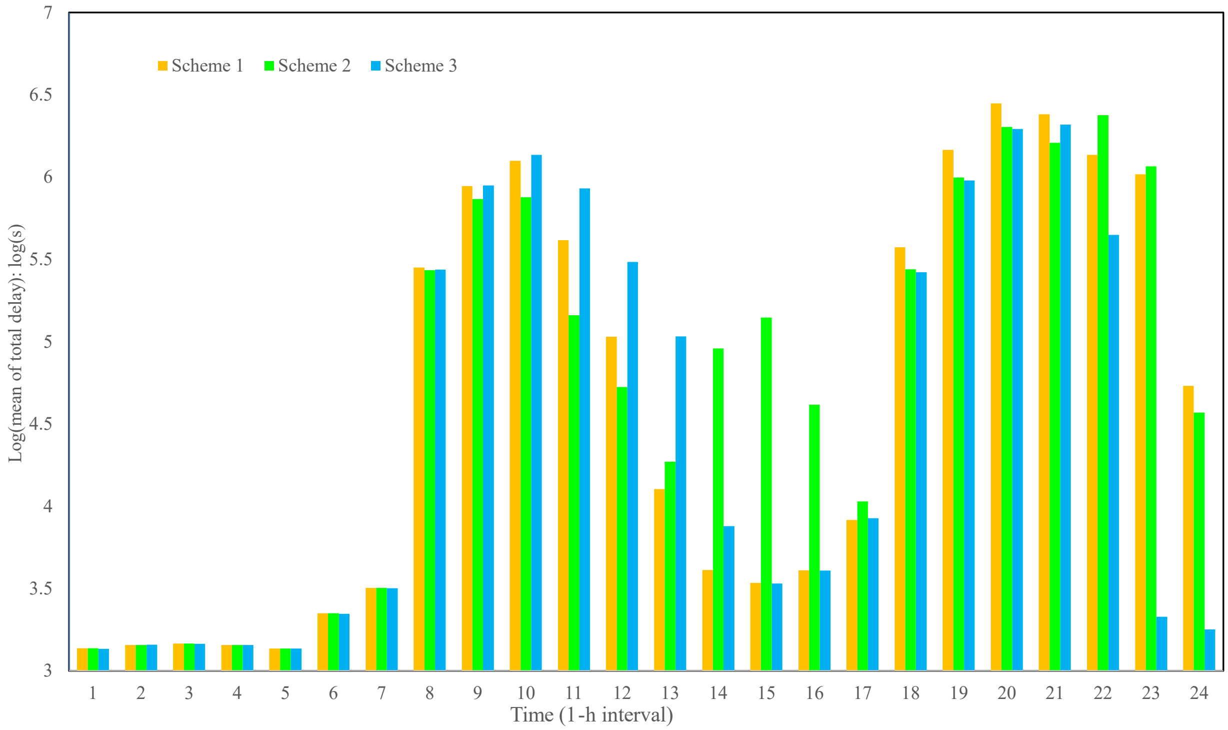
For three schemes, we collect the simulation results based on TOD benchmark partitions (**Figure 8**) for the convenience of analysis and comparison (because Scheme 1 has no real partitions and Scheme 3 has a different partition plan). From the results, we find that Scheme 3 has a lower sum of total delay and a lower sum of SD of partitional total delay than Scheme 1 and 2 with a decrease of 31% and 12.6%, 44.7% and 29.3%, respectively for the test set. However, for each interval, the mean and SD of total delays are varied, and the proposed methodology tends to preferentially minimize the mean and SD for the worst TOD intervals (i.e., interval 5 in this case). Here, Interval 5 is a typical scenario that exists residual vehicles that have not had enough time to leave at the last TOD interval. So, more reasonable TOD interval partitions with robust signal timings improve the effectiveness and the robustness from the day's perspective considering continuity.

Also, we perform a t-test to the results in the test set to validate whether the changes in daily total delay are statistically significant. For Scheme 1, we have the mean value of daily total delay as 12505479s and the SD as 906471.8s. The corresponding values of 2 and 3 are 10748214s and 665684.3s, 9546041s and 635961.6s, respectively. The degree of freedom is 18 and the confidence level is 0.05. We then calculate the test statistic for which the null hypothesis is that the two means are equal. The test statistic is 8.45 (between Scheme 1 and 3) and 4.13 (between Scheme 2 and 3) which indicate that the proposed plan improves in daily total delay based on Scheme 1 and 2.

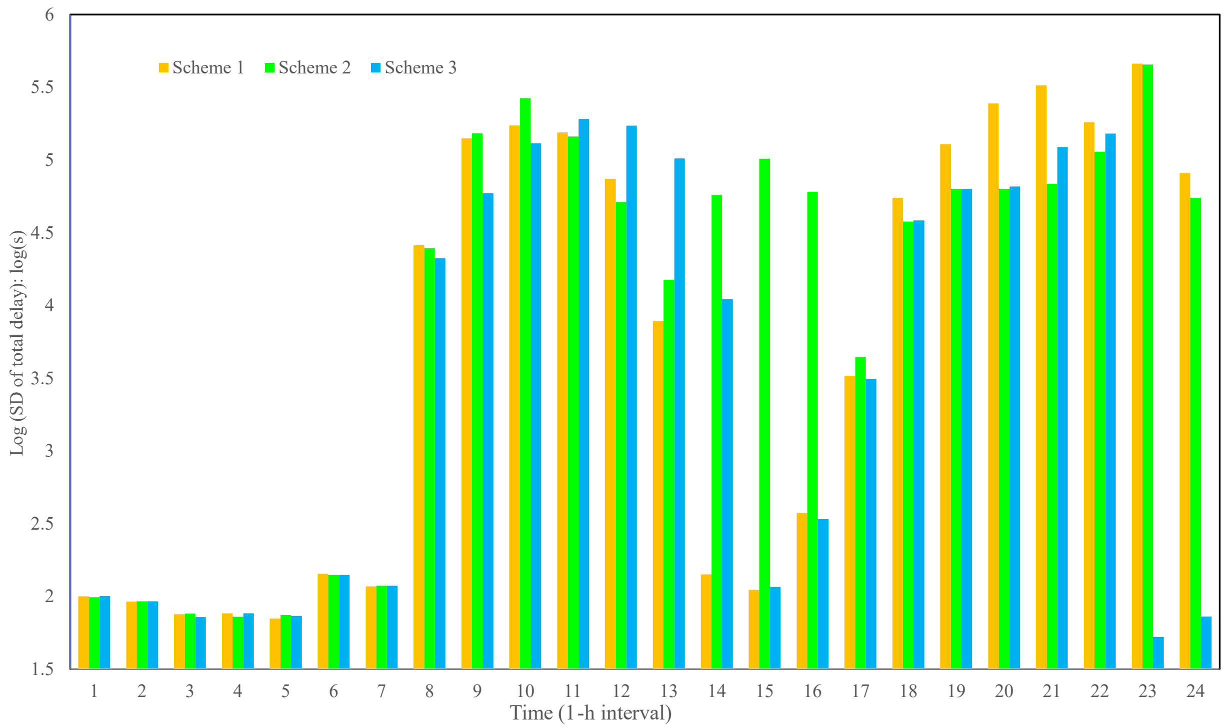
In addition, we further analyze the mean and SD of hourly total delay for test sets in the logarithmic form which improves the identifiability of the comparison of results (**Figure 9**). For the mean of hourly total delay, it can be found that the proposed plan leads to an increase in total delay from 8:00 to 13:00 and a relatively sharp decrease from 13:00 to 15:00 and from 22:00 to 24:00. For the first seven hours, three schemes share a similar performance because of the relatively low traffic demand. Particularly, the higher total delays that happen in 21:00-24:00 under Scheme 1 and 2 show the advantages of the proposed method over single plan control and TOD partition schemes that do not adequately account for flow fluctuations when dealing with residual vehicles.

And for SD, the proposed method yields better performance from 13:00 to 17:00 in test sets compared with Scheme 1 and 2, particularly sharp decreases happen from 22:00 to 24:00 which also represents the weakening robustness caused by the residual vehicles can effectively be improved by the proposed method. Also, the SDs of Scheme 3 from 10:00 to 13:00 are larger than that of Scheme 1 and 2 which indicates that the proposed plan does not yield better performance for every hour. Besides, three schemes have similar SDs from 0:00 to 7:00, which are in accordance with the mean.

In general, similar to the partitional total delays mentioned above, the proposed method tends to improve the worst-case hour's performance as a preference, and more consideration is given to delays of residual vehicles caused by unreasonable TOD partitions.



a) Mean of hourly total delay of the test set



b) SD of hourly total delay of the test set

Figure 9 Total delay performance (in logarithmic coordinate) for 24 hours of the test set

CONCLUSIONS

This paper proposes a bi-level optimization framework to jointly consider TOD interval partition and robust signal timings. The UL of the bi-level optimization is the total delay of all TOD intervals for multiple lane groups and multiple days, while the LL is the partitional mean-variance robust signal timing model. First, the delay estimation method is discussed based on CACs and CDCs and the HCM formula. Next, a TOD interval regulation method is designed that covers the conflict due to TOD interval duration not being divisible by cycle length. The evolutionary algorithm BLEAQ-II is selected to solve the bi-level optimization problem. Finally, we design a SUMO simulation platform as the validation approach. The single-plan method, the *k*-means clustering method, and the proposed method have been conducted through the platform. The results show that the clustering method and the proposed method outperform the single-plan method. And the total delay of the proposed method has been decreased by 12.6% and the robustness has been improved by 29.3% compared with the *k*-means clustering method, which shows the proposed approach can well handle the robust TOD interval partitions problem with better performance than the benchmark methods.

However, limitations are needed to be considered for future study. First, the method can be improved by considering different number of TOD intervals. Then, field tests of the proposed method can be further conducted for practical validation. In addition, further research will be focused on the robust TOD interval partition technique with the actuated signal control facilities.

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AUTHOR CONTRIBUTIONS

The authors confirm their contribution to the paper as follows: study conception and design: Chengchuan An, Zhenbo Lu, Jingxin Xia; analysis, interpretation of results, and draft manuscript preparation: Yunqing Jia. All authors reviewed the results and approved the final version of the manuscript.

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